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APPLICATION OF THE LOGARITHMIC DERIVATIVE IN ECONOMICS

SH. M. Karimova
Senior lecturer at Namangan State Technical University

A.B.Mamatxonov
Student at Namangan State Technical University

Abstract: This article examines the use of the logarithmic derivative in economics, in particular its importance in the analysis of financial investments, labor productivity, and sales. It explains how the logarithmic derivative can be used to determine the relative rate of change of economic indicators, and how this method can be used to assess interest rates, asset returns, and sales dynamics.

Keywords: Logarithmic derivative, economics, financial investments, interest rate, profitability, sales volume, growth rate, labor productivity, forecasting.

Let the utility of financial investments be the value of the investment at time t, S(t). If interest is calculated continuously, we know that over time the value of S(t) $S(t)=S_0*e^r$ is calculated by the formula. In this S_0- is the initial value of the investment, the annual nominal interest rate of bank r. Then the logarithmic derivative of S(t) is equal to:

$$(\ln S(t))'=(\ln S_0+rt)'=r$$

From this we can conclude: The annual interest rate of the bank is equal to the logarithmic derivative of the investment r. Thus, the derivative of the investment characterizes its profitability. This rule is also valid in a more general case, that is, when the interest rate changes depending on time. In this case, the logarithmic derivative of the investment represents the instantaneous profitability. Let A(t) be the value of some asset A at time t. Let r be the profitability of the investment in other assets (for example, in a bank or deposit). For simplicity, let r be independent of t. In the securities market, it is important to know when to buy and when to sell asset A. To answer this question, we need to find the instantaneous profitability of asset A that is greater than r , that is $(\ln A(t))'>r$ time interval satisfying the condition (t_1, t_2) is found. This meanst_1 purchase asset A at time and t_2 It is during this time interval that the return on asset A is higher than the return on other assets.

The law of change of the process: If y=f(x) is defined by the functional origin, the relative speed (rate) of the process is determined using the logarithmic derivative as follows:

$$T_y = (lny)' = \frac{y'}{y}$$

Example: Labor productivity of a working group y=$-2{,}5t^2 + 15 \cdot t + 100$ Let be given by the initial. Here t(0<t<8) is the working hour, calculate the change in the rate of labor productivity when t=2 and t=7 .

Solution. The rate of change in labor productivity y'=-5t+15.

The relative rate of change is $T_y = (lny)' = \frac{-5t+15}{-2{,}5t^2+15t+100}$ will be. t=2 where y'(2)=5 and $T_y = \frac{1}{24} \approx 0{,}04;$ $t = 7$ where $y'(7) = -20$ and $T_y = -\frac{8}{33} \approx -0{,}24$ it has been.

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Thus, at t=2, that is, 2 hours after the start of work, the rate of labor productivity is 5 units per hour (5 units per hour), and after 7 hours -20 units per hour, (-20 units per hour) the rate of change is (-0.24) units per hour. The plus and minus signs indicate that labor productivity increases at the beginning of the shift (at t=2), and decreases at the end of the shift (t=7). This corresponds to real reality.

Logarithmic growth rate of sales volume.

Suppose that the sales volume of products manufactured at an enterprise is determined by the function Q(t) over time. If sales volume increases continuously, it is expressed in the form:

$Q(t)=Q_0 e^{kt}$

here: $Q_0 -$ is the initial value of sales volume, k is the growth coefficient of sales volume, t is time. We find the logarithmic derivative of sales volume:

$(lnQ(t))'=(lnQ_0+kt)'=k.$

Therefore, the logarithmic derivative of sales volume is constant, which represents the instantaneous relative growth rate of product sales.

Example. Let the sales volume of a product in an enterprise be expressed as a function of time:

$Q(t)=500e^{0,04t}$,

where: Q(t)- the number of products sold in month t, t- time (in months). Determine the instantaneous relative growth rate of sales volume. Solution. First, we take the logarithmic derivative:

$T_Q=\left(lnQ(t)\right)'$

ln(Q*(t)*=ln500+0,04t

Now we can derive: (ln$Q(t))'=0,004$ it follows from this that, $T_Q=0{,}04$ sales volume is increasing by 4% every month, and this growth is considered constant. From this issue, it can be concluded that the logarithmic derivative allows us to determine not the absolute change in economic processes, but their relative rate of change. Therefore, the logarithmic derivative is of great practical importance in analyzing sales volume, assessing and forecasting economic growth rates.

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