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NATURAL GROWTH MODEL

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Abstract: *Many processes in the economy occur naturally through mechanisms of growth (multiplication). Such processes include the natural growth of the country's population in the economic system, the relationship between resources and production volumes, as well as the increase in funds deposited in the banking system for the purpose of savings over time. This article analyzes the natural growth model used in economics, its mathematical expression and economic content. The study highlights the theoretical foundations of the use of differential equations and exponential functions in describing natural growth processes and justifies their importance in modeling economic processes.*
Keywords: Natural growth process, multiplication, differential equation, integration, exponential function.

In recent decades, identifying sustainable sources of economic growth has become a central concern of both theoretical and applied research. While traditional growth models primarily emphasize capital accumulation and technological progress as the main drivers of growth, many of them fail to adequately explain the internal (endogenous) mechanisms through which economic systems evolve over time. In this context, the **Natural Growth Model** has emerged as a conceptual approach that interprets economic growth not as the result of external shocks or enforced stimuli, but as an outcome of the economy's internal potential and self-reinforcing dynamics.

The Natural Growth Model links economic growth directly to the natural circulation of resources, demographic dynamics, the evolutionary development of human capital, and the adaptability of institutional frameworks. According to this model, economic systems are capable of forming stable growth trajectories through internal coordination mechanisms without deviating from long-run equilibrium. This perspective allows growth to be evaluated not only in quantitative terms—such as GDP or output expansion—but also through qualitative dimensions, including efficiency, adaptability, and sustainability.

The purpose of this article is to examine the theoretical foundations of the Natural Growth Model, analyze its strengths and limitations in explaining economic growth, and assess its applicability in empirical research. The findings aim to contribute to economic policy design at both regional and national levels by highlighting the importance of growth strategies grounded in endogenous and naturally evolving economic factors.

Many processes observed in nature and society change (increase) by the same multiplicative factor over equal time intervals. Such processes are referred to as **natural growth processes**. Below, examples of natural growth are presented from natural and socio-economic fields.

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1. **Capital accumulation.** Funds deposited into savings increase by the same factor over the observed time period. For example, if the capital grows by 20% per year, the initial amount increases by a factor of 1.2 in the first year. In the second year, it again increases by 20%, meaning that the amount calculated in the first year grows by 1.2×0.21.2 \times 0.21.2×0.2, and the total capital becomes 1.2+1.2×0.2=1.441.2 + 1.2 \times 0.2 = 1.441.2+1.2×0.2=1.44 times the initial amount. Thus, the accumulation of savings follows the law of natural growth.
2. **Chain reactions in physics.** It is known from physics that in a chain reaction, atomic nuclei split and produce neutrons. In the observed process, the greater the number of free neutrons, the more frequently they collide with nuclei, resulting in the formation of even more new neutrons. Therefore, the increase in the number of free neutrons also represents a natural growth process.
3. **Bacterial reproduction in biology.** As an example from biology, bacteria reproduce over time; that is, each bacterium divides once per hour. This means that one bacterium becomes two bacteria in one hour. For instance, suppose there is one bacterium in a laboratory. It divides into two every hour. Accordingly, after one hour there are two bacteria, after two hours four, after three hours eight, after four hours sixteen, and so on. Hence, the process of bacterial division also follows the natural growth law.

In both nature and society, there are many processes that resemble chain reactions of reproduction. For example, in society, the growth of the human population from generation to generation, price increases, and rising inflation, as well as in nature the reproduction of animals, fish, insects, bacteria, and infectious diseases, all represent natural growth processes.

The plant known as *Kuroslep* produces 15,000 seeds three times a year; thus, in one year, $15000^3 = 3375$ billion plants could potentially germinate. Among fish species, herring lays about 30,000 eggs per year, carp more than one million, cod (*Treska*) between 4 and 6 million, sole fish (*Soliter*) about 42 million, and *Ascaris* approximately 64 million eggs. If natural selection and the struggle for survival did not exist, even if reproduction were slower than described, an unprecedented increase in the number of living organisms would occur worldwide. Calculations show that without such constraints, the descendants of a single pair of flies could, within two years, reach a total mass exceeding the mass of the Earth.

There are also well-known cases in which animals and plants, having entered favorable environmental conditions, multiplied excessively and caused disasters. Examples include locust outbreaks in Africa, rabbit overpopulation in Australia, and the spread of the Afghan bird in Uzbekistan.

If we assume that the value of the observed quantity y(t) increases by the same factor not over a finite time interval but instantaneously, then at time ttt the rate of change v(t) of the quantity is proportional to the quantity itself. In this case, the process can be expressed as

v(t) = ky(t). v(t) = y'(t)

Since v(t)=y′(t)v(t) = y'(t)v(t)=y′(t), we obtain the following differential equation:

y′(t)=ky(t).y'(t) = k y(t).y′(t)=ky(t).

This equation is called the differential equation of natural growth. It was first derived by Jacob Bernoulli in the context of credit and interest calculations.

Let us solve this equation in its general form. Separating variables in the equation

y′(t)=ky(t),y'(t) = k y(t),y′(t)=ky(t),

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and integrating, we obtain

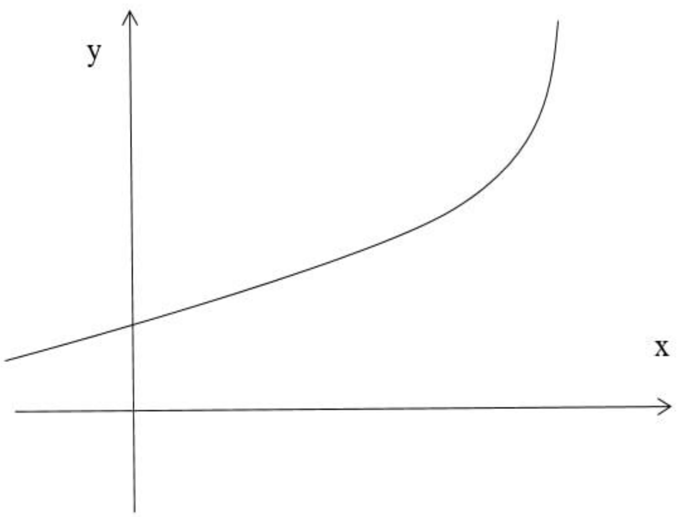
$$\frac{dy}{y} = k\,dt \;\Rightarrow\; \ln y = kt + \ln c \;\Rightarrow\; y(t) = ce^{kt}$$

Thus, the general solution of the natural growth equation (the natural growth model) is

$$y(t) = ce^{kt}$$

which is an exponential function.

$y(t_0)=y_0$, then from $y(t) = ce^{kt}$ we obtain $c = y_0e^{-kt_0}$, and therefore $\mathbf{y(t) = y_0e^{k(t-t_0)}}$



Using this model (function), it is possible to describe the dynamics of price growth when the inflation rate is constant, radioactive decay, bacterial reproduction, and other similar processes. Since the natural growth problem has been solved in its general form, in what follows we will directly use the solution

$$\mathbf{y(t) = y_0e^{k(t-t_0)}}$$

when analyzing specific natural growth problems.

It is appropriate to apply the natural growth model to the initial stages of economic system development over a limited time interval, because in the expression $y(t) = y_0e^{k(t-t_0)}$, when ttt becomes sufficiently large, the value of y(t) increases exponentially and diverges significantly from real-world behavior.

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